Numerical Study of Basketball Lay-up Shot

Research Question: What is the relationship between the angular velocity and translational velocity of the basketball in layup shots.

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Contents

1 Introduction

1.1 Background

Scientific analysis has a significant impact on sports, including data analysis, sports medicine, and training methods [1]. In basketball, besides of studying the athletes themselves—crucial for monitoring their health and maintaining strength—studying the flight of the basketball is vital for improving players' performance. Players can make better decisions about positioning and launch angles if they understand the math and science behind the game.

The academic evolution of basketball trajectory and mechanics started from broad numerical shooting models and then developed into specialized analyses of shot types, such as bank shots. Early works—such as *Numerical Analysis of the Basketball Shot* [2] and *A Theoretical Mechanics Analysis of Shooting Basketball* [3]—established comprehensive frameworks for quantifying variables like launch angle, velocity, and spin. Although these studies primarily addressed shooting problems like-free throws, their foundational insights paved the way for later investigations.

Building on these models, subsequent research acknowledged that bank shots present distinct geometric challenges due to the ball's interaction with the backboard. *Optimal Targets for the Bank Shot in Men's Basketball* [4] not only proposed a comprehensive mathematical model considering many factors like drag force, but also successfully identified optimal impact zones on the backboard. This work effectively bridged the gap between general shot mechanics and the specialized demands of bank shot analysis.

Further researches incorporated more advanced computational methods. For instance, *Application of Monte Carlo Simulations to Improve Basketball Shooting Strategy* [5] employed stochastic modeling to simulate different shot conditions, yielding probabilistic conclusions that are applicable to bank shot optimization. More recently, *LSTM-BEND: Predicting the Trajectories of Basketball* [6] used machine learning to predict ball trajectories with high precision, further refining our understanding of shot dynamics in more complicated scenarios, in which more factors are considered.

Together, these studies illustrate a clear evolution—from early general models to targeted analyses of bank shots—demonstrating how theoretical insights and computational innovations have progressively enriched the academic field while informing practical strategies in competitive basketball.

Personally, I have played basketball for several years and have reviewed numerous basketball theories and watched plenty of instructional videos to improve my shooting skills. However, most of these videos rely on the coach's personal experience rather than scientific research, and among researches an investigation to the rotation in bank shot (layup shot) is lacked temporarily. Therefore, this research will investigate "what is the relationship between the angular velocity and translational velocity of the basketball in layup shots."

1.2 Scenario

The layup, also known as a bank shot layup, is one of the most fundamental and important techniques in basketball. Players execute layups by either shooting the ball off the backboard or rolling it into the basket with their fingers. Layups are effective scoring techniques since they are the closest shots to the basket, aside from dunks. Over the past few decades, several types of layups have been developed and practiced. For example, players must impart spin to the ball at a certain velocity and shoot it at a strategic angle to avoid defenders.

Given the importance of layups in basketball, players need to increase the probability of successfully scoring. Therefore, controlling the trajectory of the ball is crucial. Brancazio proposed that players could shoot more consistently if their shots require less energy [1]. This hypothesis has been used in other related studies and will also be used in this paper. Besides controlling the velocity, players must adjust the launch angle and spin of the basketball.

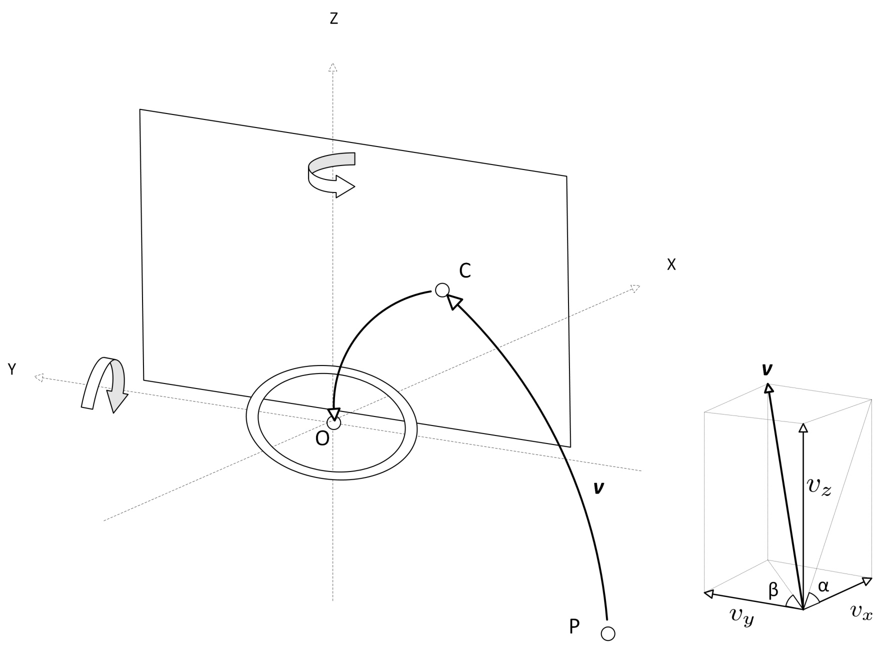


Figure 1: Figures in three dimensions

2 Assumptions

1. This paper neglects the effects of air interactions, including air resistance, buoyant force, the Magnus effect, and others. Thus, the only forces acting on the basketball are gravitational force and the normal force from the backboard. Additionally, the trajectory of the ball in the air is assumed to be a perfect parabola, which will be demonstrated in this paper.

2. A goal is considered scored when the center of the basketball coincides with the center of the basket rim.

Since the radius of the basketball is smaller than that of the basket rim, the ball will enter the basket when the centers coincide. Furthermore, the ball must enter the basket from above.

3. In practice, players perform a layup when their jump height is insufficient for a dunk. Therefore, we assume the initial height of the ball is lower than the height of the basket rim.

4. This paper uses real-world data. For example, the upper edge of the basketball rim in the NBA is 10 feet (3.05 m) above the floor and parallel to it. The mass of the basketball is approximately 0.6 kg. The coefficient of restitution is between 0.85 and 0.88, and we specify it as 0.86 [7]. The magnitude of gravitational acceleration is approximately 9.81 m/s^2.

3 Theory

3.1 Projectile trajectory function

3.1.1 Projectile Trajectory Function in Two Dimensions

Neglecting air resistance, the flight of the basketball can be treated as projectile motion, because the only forces acting on the ball are gravity and the normal force from the backboard. The basketball may also spin due to initial spin or interaction with the backboard, which will be explained later.

First, we analyze the trajectory function in two dimensions. We define the X-axis as the forward direction and the Z-axis as the vertical direction. The variable x represents the horizontal displacement, z represents the vertical displacement, $\vec{v}$ represents the initial velocity, and γ represents the shooting angle, which is the angle between the initial velocity and the X-axis. Decomposing the vector $\vec{v}$ into components along the X-axis and Z-axis, we have $v_x = |\vec{v}| \cos \gamma$ and $v_z = |\vec{v}| \sin \gamma$.

The velocity components are given by:

$$\dot{x} = v_x = |\vec{v}| \cos \gamma$$

$$\dot{z} = v_z = |\vec{v}| \sin \gamma$$

Integrating both sides, we obtain:

$$x = v_x t + c$$

Given the initial conditions $t = 0$ and $x = 0$, we find $c = 0$. Thus:

$$x = v_x t \tag{1}$$

For the Z-direction, the velocity is expressed as:

$$v_z = v_{z0} - gt$$

where v_{z0} is the initial velocity in the Z-direction, g is the gravitational acceleration, and t is the flight time. Substituting v_z with the derivative of displacement $\dot{z}$, we have the follows.

$$\dot{z} = v_{z0} - gt$$

Integrating both sides, we get:

$$z = v_{z0}t - \frac{1}{2}gt^2 + c$$

Given the initial conditions $t = 0$ and $z = h_1$, we find $c = h_1$, where h_1 is the initial height of the ball. Therefore:

$$z = v_{z0}t - \frac{1}{2}gt^2 + h_1 \tag{2}$$

To build a trajectory function, the independent variable should be x and the dependent variable should be z. Using $x = v_{x0}t$, we substitute $t = \frac{x}{v_{x0}}$ into the equation above:

$$z = v_{z0}\frac{x}{v_{x0}} - \frac{1}{2}g\left(\frac{x}{v_{x0}}\right)^2 + h_1$$

Rearranging gives:

$$z = \frac{v_{z0}}{v_{x0}}x - \frac{g}{2v_{x0}^2}x^2 + h_1$$

Substituting $v_{x0} = |\vec{v}|\cos\gamma$ and $v_{z0} = |\vec{v}|\sin\gamma$, we have:

$$z = \frac{|\vec{v}|\sin\gamma}{|\vec{v}|\cos\gamma}x - \frac{g}{2(|\vec{v}|\cos\gamma)^2}x^2 + h_1$$

Simplifying, we obtain the following.

$$z = \tan\gamma \cdot x - \frac{g}{2(|\vec{v}|\cos\gamma)^2}x^2 + h_1 \tag{3}$$

Thus, the trajectory function in two dimensions is established.

3.1.2 Projectile trajectory function in Three Dimensions

For the trajectory function in 3 dimensional space, we can express the point in three dimensional space as $R(x) = (x, y(x), z(x))$. For trajectory function in x-direction, we have the same equation with the one in two dimensions, since the displacement in x-direction will be used as an unit to build further trajectory functions. Indeed, we have:

$$x = v_x t$$

We have already yielded the function of z with respect to x from Eqns. (3). However, we still need to adjust the angle to vector.

For the function of y with respect to x, we know that there is no forces in y-direction similar with the situation in x-direction. We substitute x variables with y and we get

$$y = v_y t$$

Also, in order to build a trajectory function, the independent variable should be x and the dependent variable should be y. For the equation above, substituting t with $t = \frac{x}{v_x}$ using the equation $x = v_x t$. Then we have:

$$y = \frac{v_y}{v_x} x \tag{4}$$

In z-direction, there is gravitational force applying on the ball, so the function is different from them in x-direction and y-direction. Actually, the function of displacement in z-direction with respect to time is same with the previous one (Equation 2).

$$\begin{aligned} z &= v_{z0} t - \frac{1}{2} g t^2 + h_1 \\ &= v_{z0} \frac{x}{v_{x0}} - \frac{1}{2} g (\frac{x}{v_{x0}})^2 + h_1 \end{aligned} \tag{5}$$

To discuss easily, the velocity in three dimension will not appear; instead, the components of it—the velocity in x-direction, the velocity in y-direction, and the velocity in z-direction—will substitute it.

3.2 Bounce Theory

3.2.1 General View

There are basically three types of rebound for the basketball in two dimensional space: without initial spin, with forward initial spin, and with backward initial spin (See in Figure 2). By the way, the spin can be at any direction and with any velocity technically.

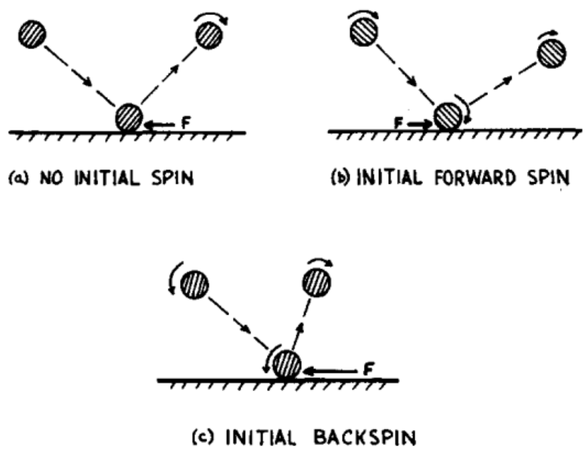
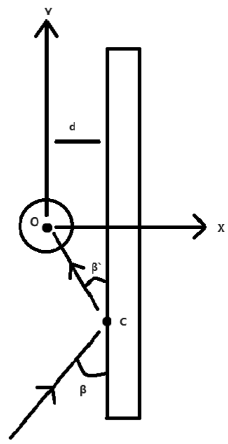
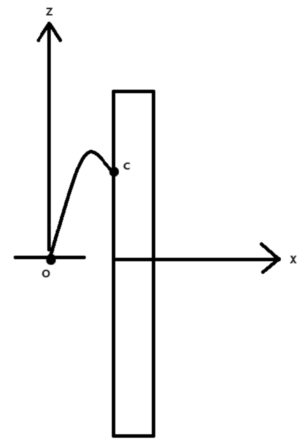


Figure 2: Brancazio, P. J. (1981) [Graph]. Physics of basketball [1]

We specify the x-direction as forward direction to the basket, the y-direction as horizontal direction along the backboard, and the z-direction as the vertical direction (See in Figure 3). The distance between the center of the basket rim and the board is d. The center of the rim is the origin point O. The point that the collision happens is named point C.



(a) Overhead view



(b) Side view

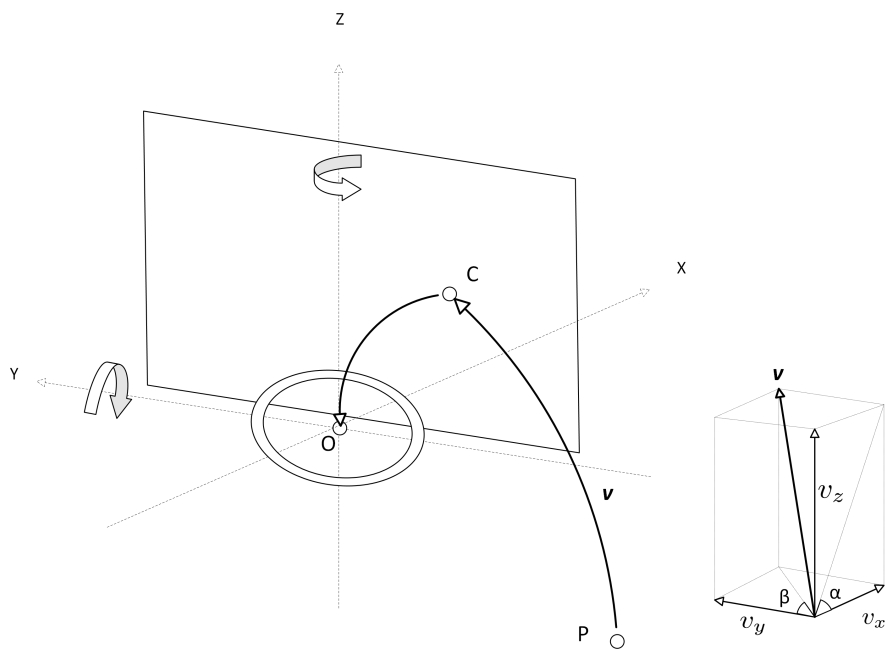


Figure 3: Rebound off the backboard

The motion of ball during the process of the lay-up shot could be classified into three different stages: the stage before the ball hits the basket board, the stage during the ball is hitting the board, and the stage when the ball leaves the basket board. The first and the last stages all can be viewed as projectile motion and they should be focus on the velocity change.

Next, we specify variables in this numerical analysis. First, the linear velocity just after the player shoot the ball is v_0, the linear velocity just before the ball collides the backboard is v_1, and the linear velocity just after collision with the backboard is v_2. Similarly, the angular velocity just after the player shoot the ball is ω_0, the angular velocity just before the ball collides the backboard is ω_1, and the angular velocity just after collision with the backboard is ω_2. Besides, the angle between the initial velocity and the x-axis is α, and the angle between the initial velocity and the y-axis is γ. The angle between the velocity v_1 and y-axis is β, and the angle between the velocity v_2 and y-axis is $\beta^{'}$. The initial point of the ball is point P_0.

Variable	**Explanation**
x	Displacement in x-direction.
y	Displacement in y-direction.
z	Displacement in z-direction.
v_0	The linear velocity just after the player shoot the ball.
v_1	The linear velocity just before the ball collides the backboard.
v_2	The linear velocity just after collision with the backboard.
ω_0	The angular velocity just after the player shoot the ball.
ω_1	The angular velocity just before the ball collides the backboard.
ω_2	The angular velocity just after collision with the backboard.
g	The magnitude of gravitational acceleration, 9.81 m/s^2.
k	The restitution coefficient in x-direction.
h_1	The initial height of the basketball, which is 1.7 m
h_2	The height of the basket rim, which is 3.05 m
d	The horizontal distance between the basket rim and the backboard.
c	The distance between the basket rim and the initial height of basketball.
α	The angle between the initial velocity and the x-y plane.
θ	The angle between the initial velocity and the y-z plane.
β	The angle between the velocity v_1 and y-axis.
$\beta^{'}$	The angle between the velocity v_2 and y-axis.
O	The original point.
C	The collision point.
P_0	The initial point.

Table 1: Variable Explanation Table

3.2.2 Simplified Bounce

To better illustrate the solution, we first consider rotation in only one dimension and motion in two dimensions. We will discuss more complex cases once the model of the bounce is clearly established. Specifically, we consider the motion in the y-direction and z-direction initially.

When the ball collides with the backboard, there is a frictional force that affects the vertical velocity of the ball, v_z, and energy loss due to the collision in the y-direction. This friction also contributes to a change in rotational velocity. With the overall loss of kinetic energy, there is energy transfer between translational and rotational modes [1]. Additionally, we do not consider sliding of the ball. The frictional force acts in the direction opposite to the movement, causing the ball to spin along the z-axis, as demonstrated below.

We define the momentum as a vector $\vec{P}$, which represents the product of the mass of the ball and its linear velocity $\vec{v}$. The change in momentum, called impulse, is given by $J = \Delta\vec{P} = m\Delta\vec{v}$.

According to rebound theory, after the collision, the velocity changes direction, and its magnitude is equal to the original magnitude multiplied by a constant k, which is the restitution coefficient in the y-direction. The equation for the velocity after collision is:

$$v_x = -kv_{x0} \tag{6}$$

Decomposing the impulse into the y-direction and z-direction, we have:

$$\begin{cases} J_y = m\Delta v_y \\ J_z = m\Delta v_z \end{cases} \tag{7}$$

We define the angular velocity resulting from friction as ω. Since the ball has no initial angular velocity, a single variable is sufficient to represent it. The moment of inertia, I, describes how difficult it is to rotate an object. More specifically, $I = \sum mr^2$, where the moment of inertia is the sum of each part's contribution. For a thin spherical shell like a basketball, $I = \frac{2}{3}mr^2$, where m is the mass and r is the radius [8]. The angular momentum of the ball, denoted by L, is the product of the moment of inertia and the angular velocity [8]. The change in angular momentum, known as angular impulse, is given by:

$$\Delta L = I(\omega_2 - \omega_1) \tag{8}$$

Since the initial angular velocity is zero, $\Delta\omega = \omega$.

According to the impulse-momentum principle, $\Delta L = \tau\Delta t$. Rearranging gives $\Delta L = Fr\Delta t$. Since impulse is the product of force and time, we have $J = F\Delta t$. Therefore, for angular impulse:

$$\Delta L = Jr \tag{9}$$

By equating equations (8) and (9), we obtain:

$$I(\omega_2 - \omega_1) = Jr \tag{10}$$

Next, by equating the second equation in (7) with (10) and substituting ω with ω_y and J with J_z, we have:

$$I(\omega_{y2} - \omega_{y1}) = m\Delta v_z r = m(v_{z2} - v_{z1})r \tag{11}$$

Notice that angular impulse in y-direction will be converted to the impulse in z-direction, since the angular impulse vector is perpendicular to the plane of rotation and indicates the axis around which the rotation occurs, according to the definition of vectors related to angle.

Since we do not consider sliding, the final velocity in the z-direction is:

$$v_{z2} = \omega_{y2}r \tag{12}$$

$$v_{y2} = -\omega_{z2} r \tag{13}$$

By substituting $v_{z2} = \omega_{y2} r$ in equation (11), we get:

$$\frac{I(\omega_{y2} - \omega_{y1})}{r} = m(\omega_{y2} r - v_{z1}) \tag{14}$$

$$\frac{I(\omega_{z2} - \omega_{z1})}{r} = m(-\omega_{z2} r - v_{y1}) \tag{15}$$

Rearranging gives:

$$\omega_{y2}\left(\frac{I}{r} - mr\right) = -mv_{z1} + \frac{I}{r}\omega_{y1} \tag{16}$$

$$\omega_{z2}\left(\frac{I}{r} + mr\right) = -mv_{y1} + \frac{I}{r}\omega_{z1} \tag{17}$$

By substituting $I = \frac{2}{3}mr^2$, we have:

$$\omega_{y2}\left(\frac{2}{3}mr - mr\right) = -mv_{z1} + \frac{2mr}{3}\omega_{y1} \tag{18}$$

$$\omega_{z2}\left(\frac{2}{3}mr + mr\right) = -mv_{y1} + \frac{2mr}{3}\omega_{z1} \tag{19}$$

Simplifying:

$$\omega_{y2} = \frac{3(v_{z1} - 2r\omega_{y1})}{r} \tag{20}$$

$$\omega_{z2} = \frac{3(-v_{y1} + 2r\omega_{z1})}{5r} \tag{21}$$

We have now obtained the angular velocity after the collision, which depends on three factors: the initial linear velocity, the radius, and the initial angular velocity. Although there is no further interaction involving this angular velocity in the paper, it plays a significant role in contributing to the ball's trajectory due to the Magnus effect. In addition to the angular velocity, we need to calculate the final linear velocity.

By equating the angular impulse and the moment of momentum from equation (9), we have:

$$I(\omega_{y2} - \omega_{y1}) = m(v_{z2} - v_{z1})r \tag{22}$$

Since the ball does not slide on the backboard, the final velocity can be expressed as:

$$v_{z2} = \omega_{y2} r \tag{23}$$

Combining these equations:

$$v_{z2} = \frac{r(mrv_{z1} - I\omega_{y1})}{I + mr^2} \tag{24}$$

Thus, we have the final angular and linear velocities after the collision.

$$\omega_{y2} = \frac{3(v_{z1} + 2r\omega_{y1})}{5r} \tag{25}$$

$$v_{z2} = 3(v_{z1} - 2r\omega_{y1}) \tag{26}$$

3.3 Bounce in Three Dimensions

Similarly to the bounce motion in two dimensions, the motion of the ball in three dimensions can be viewed to experience three stages: the stage before the ball hits the basket board, the stage during the ball is hitting the board, and the stage when the ball leaves the basket board.

3.3.1 The First Stage

Before analysis which includes angles, we first specify the velocity vector in three directions:

$$\begin{cases} v_x = v\cos\alpha\sin\beta \\ v_y = v\cos\alpha\cos\beta \\ v_z = v\sin\alpha \end{cases} \tag{27}$$

Recalling from previous equations about trajectory functions (See in Equations 1, Equations 4, Equations 5), we have:

$$\begin{cases} x = v_x t \\ y = v_y t = \dfrac{v_y}{v_x}x \\ z = v_z\dfrac{x}{v_x} - \dfrac{1}{2}g(\dfrac{x}{v_x})^2 + h_1 \end{cases} \tag{28}$$

However, we need to consider the initial point of the ball. Therefore, we add an initial constant at the end of each equation.

$$\begin{cases} x = v_x t + P_{0x} \\ y = v_y t + P_{0y} = \dfrac{v_y(x - P_{0x})}{v_x} + P_{0y} \\ z = v_z\dfrac{(x - P_{0x})}{v_x} - \dfrac{1}{2}g(\dfrac{x - P_{0x}}{v_x})^2 + P_{0z} \end{cases} \tag{29}$$

Substituting x with C_x, y with C_y, and z with C_z, we can express the collision point, which is the initial point of the second stage:

$$\begin{cases} C_x = v_{x0}t + P_{0x} \\ C_y = \dfrac{v_{y0}(C_x - P_{0x})}{v_{x0}} + P_{0y} \\ C_z = v_{z0}\dfrac{(C_x - P_{0x})}{v_{x0}} - \dfrac{1}{2}g(\dfrac{C_x - P_{0x}}{v_{x0}})^2 + P_{0z} \end{cases} \tag{30}$$

It is of importance to notice that C_x is constant, because the distance from the basket board to the center of rim, original point O, is constant in x-axis. Then, the launching angle can be expressed with the velocity and triangle functions.

$$\begin{cases} \alpha = \arctan(\dfrac{v_{z0}}{v_{x0}}) \\ \beta = \arctan(\dfrac{v_{x0}}{v_{y0}}) \end{cases} \tag{31}$$

Then, we can express Equation 30 just using the vector component in the x-direction v_x instead of the vector components in three directions. This is because expressing vectors with one vector component and two angles is more intuitive than expressing vectors in three vector components. In other words, players know their initial shooting angle better than vector components of their initial shooting velocity.

$$\begin{cases} C_x = v_{x0}t + P_{0x} \\ C_y = \dfrac{(C_x - P_{0x})}{\tan\beta} + P_{0y} \\ C_z = \dfrac{(C_x - P_{0x})}{\tan\alpha} - \dfrac{1}{2}g(\dfrac{C_x - P_{0x}}{v_{x0}})^2 + P_{0z} \end{cases} \tag{32}$$

In this way, it is more convenient for us to develop the system in the following passage.

3.3.2 The Second Stage

The moment of momentum can be expressed as follows:

$$\begin{cases} J_x = m\Delta v_x \\ J_y = m\Delta v_y \\ J_z = m\Delta v_z \end{cases} \tag{33}$$

According to the scenario about the actual spin, we have [7]:

$$
\begin{cases}
I(\omega_{x2} - \omega_{x1}) = 0 \\
I(\omega_{y2} - \omega_{y1}) = J_z r \\
I(\omega_{z2} - \omega_{z1}) = J_y r
\end{cases} \tag{34}
$$

The velocity can be expressed as following. Notice that $v_{x1} = v_{x0}$, since there is no external force during the whole motion.

$$
\begin{cases}
v_{x2} = -k v_{x1} \\
v_{y2} = -r\omega_{z2} \\
v_{z2} = r\omega_{y2}
\end{cases} \tag{35}
$$

According to equation 24, combining equation 33, equation 34, and equation 35, we have:

$$
\begin{cases}
v_{x2} = -k v_{x1} \\
v_{y2} = \dfrac{r(mrv_{y1} - I\omega_{z1})}{I + mr^2} \\
v_{z2} = \dfrac{r(mrv_{z1} + I\omega_{y1})}{I + mr^2}
\end{cases} \tag{36}
$$

Substituting I with $\frac{2}{3}mr^2$, we have:

$$
\begin{cases}
v_{x2} = -k v_{x1} \\
v_{y2} = \dfrac{3v_{y1} - 2r\omega_{z1}}{5} \\
v_{z2} = 3v_{z1} - 2r\omega_{y1}
\end{cases} \tag{37}
$$

The final angular velocity has no influence on the motion of the ball after the collision, so we do not discuss it here.

3.3.3 The Third Stage

After the collision, the movement is same with the one at the first stage (see in equation 28). However, in this stage, vector components should be expressed in three-components form, because we directly derive the value of vector components in the second stage and it is unnecessary to calculate angles. Through combining the final point and the collision point with the trajectory functions, we have the following.

$$
\begin{cases}
0 = v_{x2}t + C_x \\
0 = \dfrac{v_{y2}(0 - C_x)}{v_{x2}} + C_y \\
0 = v_{z2}\dfrac{(0 - C_x)}{v_{x2}} - \dfrac{1}{2}g(\dfrac{(0 - C_x)}{v_{x2}})^2 + C_z
\end{cases} \tag{38}
$$

3.4 Combination

Combine equations from the three stages above and simply:

$$\begin{cases} 0 = -kv_{x1}t + C_x \\ 0 = \dfrac{\left(\frac{3v_{y1}-2r\omega_{z1}}{5}\right)C_x}{kv_{x1}} + C_y \\ 0 = (3v_{z1} - 2r\omega_{y1})\dfrac{C_x}{kv_{x1}} - \dfrac{1}{2}g\left(\dfrac{-C_x}{-kv_{x1}}\right)^2 + C_z \end{cases}$$

Rearranging by substituting $\frac{v_{y1}}{v_{x1}}$ with $\frac{1}{\tan\beta}$ (Equation 31), we have:

$$\begin{cases} 0 = -kv_{x1}t + C_x \\ 0 = \dfrac{3C_x}{5k\tan\beta} - \dfrac{2rC_x\omega_{z1}}{5kv_{x1}} + C_y \\ 0 = \dfrac{3C_x\tan\alpha}{k} - \dfrac{2rC_x\omega_{y1}}{kv_{x1}} - \dfrac{1}{2}g\left(\dfrac{-C_x}{-kv_{x1}}\right)^2 + C_z \end{cases}$$

Here, according to Equation 32, we have $\tan\beta = \frac{C_x-P_{0x}}{C_y-P_{0y}}$ and $\tan\alpha = \frac{(C_x-P_{0x})}{C_z-P_{0z}+\frac{1}{2}g\left(\frac{C_x-P_{0x}}{v_{x0}}\right)^2}$

$$\begin{cases} 0 = -kv_{x1}t + C_x \\ 0 = \dfrac{3C_x}{5k\left(\frac{C_x-P_{0x}}{C_y-P_{0y}}\right)} - \dfrac{2rC_x\omega_{z1}}{5kv_{x1}} + C_y \\ 0 = \dfrac{3C_x(C_x-P_{0x})}{k\left(C_z-P_{0z}+\frac{1}{2}g\left(\frac{C_x-P_{0x}}{v_{x0}}\right)^2\right)} - \dfrac{2rC_x\omega_{y1}}{kv_{x1}} - \dfrac{1}{2}g\left(\dfrac{-C_x}{-kv_{x1}}\right)^2 + C_z \end{cases}$$

Through rearranging, we derive the final equations:

$$\begin{cases} 0 = -kv_{x1}t + C_x \\ \omega_{z1} = \left(\dfrac{5kv_{x1}}{2rC_x}\right)\left(\dfrac{3C_x(C_y-P_{0y})}{5k(C_x-P_{0x})} + C_y\right) \\ \omega_{y1} = \left(\dfrac{kv_{x1}}{2rC_x}\right)\left(\dfrac{3C_x(C_x-P_{0x})}{k\left(C_z-P_{0z}+\frac{1}{2}g\left(\frac{C_x-P_{0x}}{v_{x1}}\right)^2\right)} - \dfrac{1}{2}g\left(\dfrac{-C_x}{-kv_{x1}}\right)^2 + C_z\right) \end{cases} \tag{39}$$

4 Results

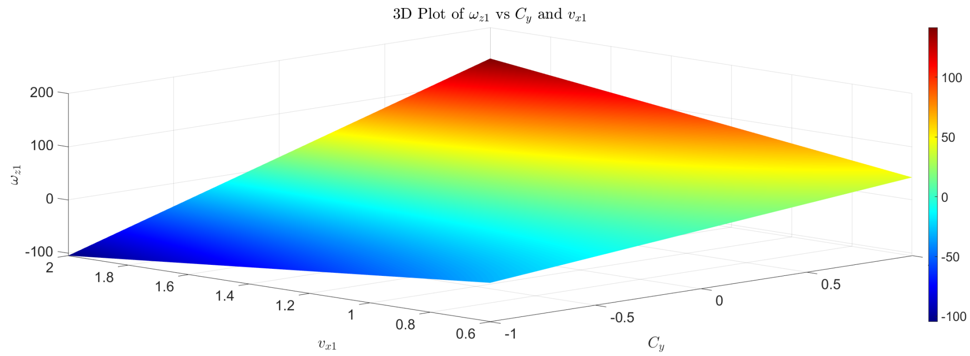


Figure 4: 3D Plot of ω_{z1} vs C_y and v_{x1}

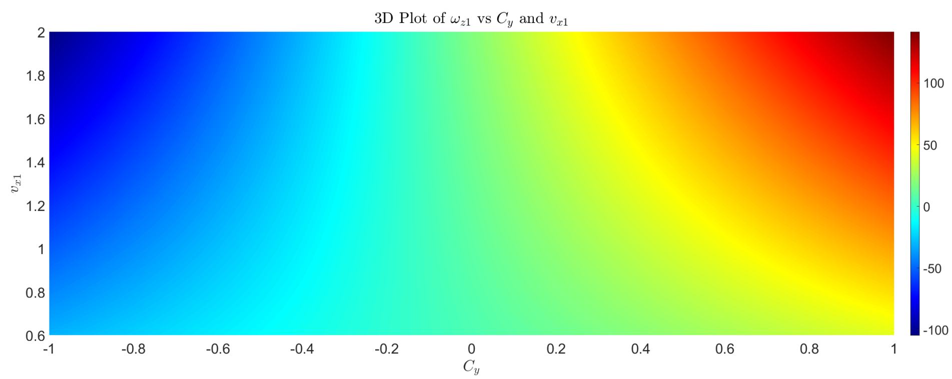


Figure 5: 3D Plot of ω_{z1} vs C_y and v_{x1}

From the equation (39), we know that for a certain C_y the angular velocity ω_{z1} is linear with the translational velocity v_{x1}. Therefore, if a player aims at a specific position on the backboard, he (she) can determine the needed angular velocity in z-direction based on the translational velocity in x-direction. The direct linear relationship is quite simple for players to adjust their layup shooting strategies on the court. From the graph depicting ω_{z1} versus C_y and v_{x1}, we observe that the lower the velocity in the x-direction, the more gradual the change in ω_{z1} with respect to C_y. This conclusion aligns with practical experience: lower velocities make it easier for the player to control the ball, reducing the adjustments needed.

In addition, for a constant translational velocity v_{x1}, the relationship between ω_{z1} and C_y is an inverse proportional function; hence, the range of C_y should be as near to 0 as possible to make the needed angular velocity small, thus increasing controlling easiness and shooting accuracy. This relationship warns players that if they choose an extreme or odd position as a shoot aim, they need to dramatically adjust the angular velocity of the ball in z-direction.

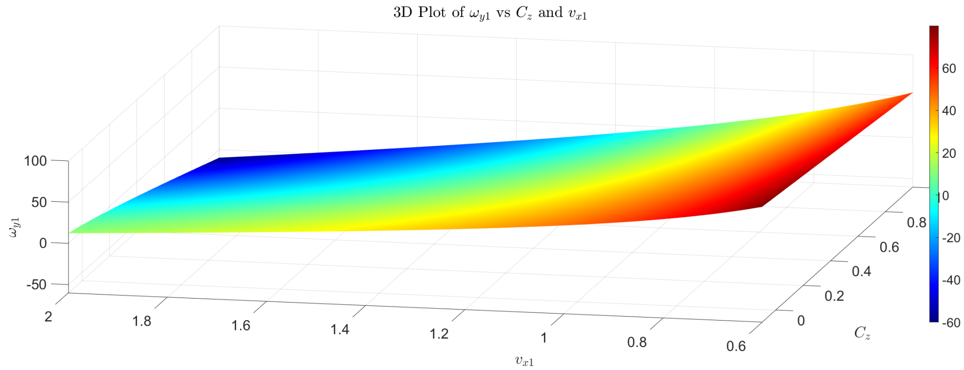


Figure 6: 3D Plot of ω_{y1} vs C_z and v_{x1}

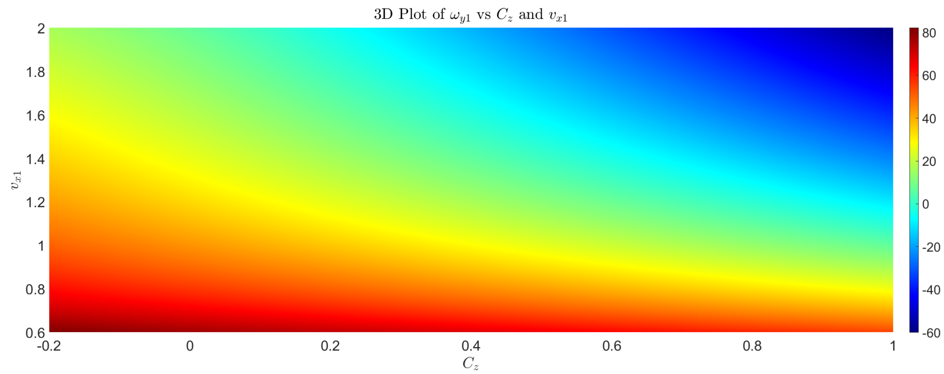


Figure 7: 3D Plot of ω_{y1} vs C_z and v_{x1}

For the graph of ω_{y1} versus C_z and v_{x1}, a similar analysis method could be implemented. The third equation for angular velocity ω_{y1} is more complicated. Intuitively, a lower x-direction velocity results in a less pronounced change in ω_{y1} with respect to C_z, making control easier. This finding is consistent with practical experience, as controlling the ball becomes simpler when the initial speed is lower.

Moreover, starting from the equation (39), we find that for a given v_{x1} the angular velocity ω_{y1} is proportional to $\frac{K}{C_z+B} + KC_z$. As the increasing of C_z, $\frac{K}{C_z+B}$ has less enough influence to ω_{y1}. However, it could not be neglected here, since discussing range of C_z is quite small (less than 1) and the difference between K and B is not big. Therefore, the relationship between C_z and v_{x1} is similar with a milder linear function.

5 Conclusion

This paper develops several mathematical models for basketball layup shots. To simplify the research question, air interactions, including air resistance, the Magnus effect, and buoyant force, are neglected. The study begins by investigating projectile motion, followed by an analysis of the rebound mechanism off the backboard. Physical theories are used to support the mathematical models, and the analysis of two-dimensional motion aids in clearly expressing the theory in three dimensions.

In Sections 3.3 and 3.4, coherent and comprehensive models are constructed to explain the relationships among various variables, such as initial angular velocity, launch angle, and initial position. It is essential to note that a direct equation relating initial angular velocity and launch angle helps to illustrate these relationships. The above results demonstrate these connections.

However, it is impractical for basketball players to recall mathematical equations and perform calculations during the game or layup practice. This is why derivatives are used: it is more important to understand how relationships change rather than memorizing specific values.

Although the conclusions may not be broadly applicable, the patterns shown in the model are still meaningful to players, as they provide general guidance for adjustments in training. The model itself is theoretical, and thus the conclusions are somewhat biased due to neglected factors, such as air resistance and the Magnus effect. Nevertheless, the pattern reveals more than just correlations; it provides a new understanding of basketball aerodynamics. Players may realize that some factors, such as the relationship between ω_y and v_0, have minimal impact, while others, such as shooting angles, play a significant role.

In practice, a general understanding of the science behind the sport can help players improve more efficiently. With numerical analysis, players are not solely reliant on empirical intuition during practice; instead, they can understand every detail of the ball's motion during a layup.

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